Unit 6: Matrix decomposition

Juan Luis Melero and Eduardo Eyras

October 2018
## Contents

1. Matrix decomposition  
   1.1. Definitions ................................................. 3

2. Singular Value Decomposition  

3. Exercises  

4. R practical  
   4.1. SVD .......................................................... 15
1 Matrix decomposition

1.1 Definitions

Matrix decomposition consists in transforming a matrix into a product of other matrices. Matrix diagonalization is a special case of decomposition and is also called diagonal (eigen) decomposition of a matrix.

**Definition:** there is a diagonal decomposition of a square matrix $A$ if we can write

$$A = UDU^{-1}$$

Where:

- $D$ is a diagonal matrix
- The diagonal elements of $D$ are the eigenvalues of $A$
- Vector columns of $U$ are the eigenvectors of $A$.

For symmetric matrices there is a special decomposition:

**Definition:** given a symmetric matrix $A$ (i.e. $A^T = A$), there is a unique decomposition of the form

$$A = UDU^T$$

Where

- $U$ is an orthonormal matrix
- Vector columns of $U$ are the unit-norm orthogonal eigenvectors of $A$
- $D$ is a diagonal matrix formed by the eigenvalues of $A$.

This special decomposition is known as **spectral decomposition**.

**Definition:** An orthonormal matrix is a square matrix whose columns and row vectors are orthogonal unit vectors (orthonormal vectors).

Orthonormal matrices have the property that their transposed matrix is the inverse matrix.
**Proposition:** if a matrix $Q$ is orthonormal, then $Q^T = Q^{-1}$

Proof: consider the row vectors in a matrix $Q$:

$$Q = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \text{ and } Q^T = (u_1|\ldots|u_n)$$

$$QQ^T = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} (u_1|\ldots|u_n) = \begin{pmatrix} \langle u_1, u_1 \rangle & \ldots & \langle u_1, u_n \rangle \\ \vdots & \ddots & \vdots \\ \langle u_n, u_1 \rangle & \ldots & \langle u_n, u_n \rangle \end{pmatrix} = \begin{pmatrix} 1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 1 \end{pmatrix}$$

Remember that:

- Orthogonal vectors $\langle u_i, u_j \rangle = 0$, $\forall i \neq j$
- Unit vectors $\langle u_i, u_i \rangle = 1$, $\forall i$

In addition, the determinant of $Q$ is $\pm 1$.

**Theorem:** if $Q$ is an orthonormal matrix $\rightarrow \det(Q) = \pm 1$.

Proof:

$$\det(Q)^2 = \det(Q)\det(Q^T) = \det(QQ^T) = \det(I_n) = 1 \rightarrow \det(Q) = \sqrt{1} = \pm 1$$

**Proposition:** The scalar product of two vectors is invariant under the transformation by a an orthonormal matrix:

$$\langle Qu, Qv \rangle = \langle u, v \rangle$$

Proof:

$$\langle Qu, Qv \rangle = (Qu)^TQv = u^TQ^TQv = u^Tv = \langle u, v \rangle$$

**Corollary:** the column (or row) vectors from an orthonormal matrix form an (orthonormal) basis of $\mathbb{R}^n$

**Example:** Perform the spectral decomposition of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. 

4
We calculate the eigenvalues and the eigenvectors:

\[
\det(A - \lambda I_2) = \det \begin{pmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{pmatrix} = \lambda^2 - 4\lambda + 3
\]

\[
\det(A - \lambda I_2) = 0 \rightarrow \lambda^2 - 4\lambda + 3 = (\lambda - 1)(\lambda - 3) = 0 \rightarrow \lambda = 1, 3
\]

\[
\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3 \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{cases} 2x + y = 3x \\ x + 2y = 3y \end{cases} \rightarrow x = y \rightarrow \begin{pmatrix} x \\ x \end{pmatrix} \text{ Eigenvectors of 3}
\]

\[
\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{cases} 2x + y = x \\ x + 2y = y \end{cases} \rightarrow -x = y \rightarrow \begin{pmatrix} x \\ -x \end{pmatrix} \text{ Eigenvectors of 1}
\]

We have the following eigenspaces:

\[
E(3) = Ker(A - 3I_2) = \left\{ \begin{pmatrix} a \\ a \end{pmatrix}, a \in \mathbb{R} \right\} \subset \mathbb{R}^2
\]

\[
E(1) = Ker(A - I_2) = \left\{ \begin{pmatrix} b \\ -b \end{pmatrix}, b \in \mathbb{R} \right\} \subset \mathbb{R}^2
\]

We have to choose two orthogonal unit vectors. The eigenvectors are already orthogonal to each other:

\[
\begin{pmatrix} a & a \end{pmatrix} \begin{pmatrix} b \\ -b \end{pmatrix} = ab - ba = 0
\]

We normalize both vectors to 1:

\[
\begin{pmatrix} a & a \end{pmatrix} \begin{pmatrix} a \\ a \end{pmatrix} = 1 \rightarrow a = \frac{1}{\sqrt{2}}
\]

\[
\begin{pmatrix} b & -b \end{pmatrix} \begin{pmatrix} b \\ -b \end{pmatrix} = 1 \rightarrow b = \frac{1}{\sqrt{2}}
\]

So the basis \(B = \left\{ \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \right\} \) is an orthonormal basis of \(\mathbb{R}^2\)

With these orthonormal vectors we build the orthonormal matrix that diagonalize \(A\):

\[
U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad U^{-1} = U^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}
\]
The matrix $U$ provides a unique diagonal spectral decomposition:

$$A = UDU^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Notice that the order of the eigenvectors in the orthonormal matrix determines the order of the eigenvalues in the diagonal matrix.

If the matrix $A$ is non-singular ($\det(A) \neq 0$) $A \in M_{n \times n}(\mathbb{R})$, we can define the inverse of $A$ using the spectral decomposition $A = UDU^T$ as:

$$A^{-1} = (UDU^T)^{-1} = (U^T)^{-1}D^{-1}U^{-1} = UD^{-1}U^T$$

where we have used that $U$ is an orthonormal matrix (built with the eigenvalues of $A$) for which the inverse is the transpose. Here $D$ is the diagonal matrix with the eigenvalues of $A$, hence $D^{-1}$ is a diagonal matrix containing the inverse of the eigenvalues. We confirm that this is the inverse of $A$:

$$AA^{-1} = UDU^TUD^{-1}U^T = UDD^{-1}U^T = UU^T = I_n$$

**Example:** Consider the same matrix as before

$$A = UDU^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

You can easily check that $AA^{-1} = I_2$.

## 2 Singular Value Decomposition

Singular Value Decomposition (SVD) is a form of dimensional reduction. SVD allows to identify the dimensions that are most relevant. Once these dimensions are identified, we can find the best approximation to the original data using fewer dimensions.

SVD generalizes the symmetrical orthonormal decomposition to non-symmetric matrices and to rectangular (non-square) matrices.
A is the input matrix of dimensions $m \times n$, $U$ is an orthonormal matrix of dimensions $m \times m$, $\Sigma$ contains a diagonal submatrix of size $\min(m, n)$ and the rest of the elements are 0, $V$ is an orthonormal matrix $n \times n$.

**Definition** of SVD: Given $A$ a rectangular matrix $A \in M_{m \times n}(\mathbb{R})$, we can find a decomposition:

$$A = U \Sigma V^T$$

Where:

- $U$ and $V$ are orthonormal matrices
- The columns of $U$ are orthonormal eigenvectors of $AA^T \in M_{m \times m}(\mathbb{R})$. It is the orthonormal matrix that diagonalizes $AA^T$. These are called the **left singular vectors** of $A$
- The columns of $V$ are orthonormal eigenvectors of $A^T A \in M_{n \times n}(\mathbb{R})$. It is the orthonormal matrix that diagonalizes $A^T A$. These are called the **right singular vectors** of $A$
- $\Sigma$ is a rectangular block-diagonal matrix containing the **square-roots** of the eigenvalues of $A^T A$ (which are the same as $AA^T$). These are called the **singular values**

Let’s try to understand where does $A = U \Sigma V^T$ come from and how to do the decomposition.

The idea is to build symmetric matrices because they have the property that the eigenvectors matrix is orthonormal. To build symmetric matrices we multiply the matrix by its transposed:

$$AA^T \in M_{m \times m}(\mathbb{R}) \quad A^T A \in M_{n \times n}(\mathbb{R})$$
These matrices are square and symmetric:

\((AA^T)^T = (A^T)^T A^T = AA^T\)  \((A^T A)^T = A^T (A^T)^T = A^T A\)

\(A^T A\) is a square symmetric matrix, so it has a set orthonormal eigenvectors.

\(V\) is the matrix built from these eigenvectors that diagonalizes \(A^T A\):

\(A^T A = (U \Sigma V^T)^T U \Sigma V^T = V \Sigma^T U^T U \Sigma V^T = V \Sigma^T \Sigma V^T = V \Sigma^2 V^T\)

So \(\Sigma\) contains the square root of the eigenvalues of \(A^T A\):

\((A^T A) V = V \Sigma^2\)

Similar for \(U\):

\(AA^T\) is a square symmetric matrix, so it has also a set of orthogonal eigenvectors.

\(AA^T = U \Sigma V^T (U \Sigma V^T)^T = U \Sigma V^T \Sigma^T U^T = U \Sigma^2 U^T\)

\((AA^T) U = U \Sigma^2\)

\(U\) is the matrix that diagonalizes \(AA^T\) and \(\Sigma\) contains the square root of the eigenvalues of \(AA^T\).

\(AA^T\) and \(A^T A\) have non-negative eigenvalues (\(\Sigma\) matrix contains squared numbers) and they coincide:

**Proposition:** \(A^T A\) and \(AA^T\) share \(\min(m, n)\) eigenvalues.
Proof: consider $v$ an eigenvector of $A^T A$:

$$A^T A = V \Sigma^2 V^T$$

$$(A^T A)v = \lambda v \rightarrow A(A^T A)v = \lambda Av \rightarrow (AA^T)Av = \lambda Av$$

So, if $v$ is an eigenvalue of $A^T A$ (in $V$) with eigenvalue $\lambda$, then $Av$ is an eigenvector of $AA^T$ with eigenvalue $\lambda$. $v$ is a column vector of $V$, hence $\langle v, v \rangle = 1$. However, for $Av$:

$$\langle Av, Av \rangle = (Av)^T Av = v^T (A^T A)v = v^T \lambda v = \lambda \langle v, v \rangle = \lambda$$

The eigenvectors of $AA^T$ in $U$ corresponding to the same eigenvalues can thus be built as:

$$u = \frac{1}{\sqrt{\lambda}} Av$$

This definition ensures that $\langle u, u \rangle = \langle Av, Av \rangle = 1$. In addition, it establishes a relationship between the eigenvectors.

As the spectral decomposition, we can use SVD to define an inverse matrix. Given a non-singular non-symmetrical matrix $A$ ($\det(A) \neq 0$) $A \in M_{n \times n}(\mathbb{R})$, we can write: $A = UDV^T$ where $U$ and $V$ are the matrix built as before, where:

- $U$ is the orthonormal matrix that diagonalizes $AA^T$
- $V$ is the orthonormal matrix that diagonalizes $A^T A$
- $D^{-1}$ is a diagonal matrix containing the reciprocal of the eigenvalues $(\frac{1}{\lambda})$ of $A$
We can define the inverse of a matrix with:

\[ A^{-1} = VD^{-1}U^T \]

\[ AA^{-1} = UDV^TVD^{-1}U^T = UDD^{-1}U^T = UU^T = I_n \]

**Example of SVD:** calculate SVD for the matrix \( \begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix} \) (\( A = U\Sigma V^T \))

We calculate \( V \), the orthonormal matrix that diagonalizes \( A^T A \)

\[ A^T A = \begin{pmatrix} 4 & -3 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} 25 & 7 \\ 7 & 25 \end{pmatrix} \]

We can see numerically here that \( A^T A \) is symmetric. Calculate the eigenvalues and matrix \( V \).

\[ \det(A^T A - \lambda I_2) = \det \begin{pmatrix} 25 - \lambda & 7 \\ 7 & 25 - \lambda \end{pmatrix} = (25 - \lambda)^2 - 7^2 = 0 \rightarrow \lambda = 32, 18 \]

\[ E(32) = \text{Ker}(A^T A - 32I_2) = \text{Ker} \begin{pmatrix} -7 & 7 \\ 7 & -7 \end{pmatrix} = \left\{ \begin{pmatrix} a \\ a \end{pmatrix} , a \in \mathbb{R} \right\} \]

\[ E(18) = \text{Ker}(A^T A - 18I_2) = \text{Ker} \begin{pmatrix} 7 & 7 \\ 7 & 7 \end{pmatrix} = \left\{ \begin{pmatrix} a \\ -a \end{pmatrix} , a \in \mathbb{R} \right\} \]

Orthonormal vectors from these subspaces (an orthonormal basis):

\[ v_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} , v_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]

\[ V = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \]

Now we can decompose the matrix:

\[ A^T A = \begin{pmatrix} 25 & 7 \\ 7 & 25 \end{pmatrix} = V \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 32 & 0 \\ 0 & 18 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \]
We can also already build the Σ matrix, since it is the matrix composed by the square root of the eigenvalues, which are shared between \( AA^T \) and \( A^T A \):

\[
Σ = \begin{pmatrix} 4\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}
\]

We do the same to build the matrix \( U \). \( U \) is the orthonormal matrix that diagonalizes \( AA^T \).

\[
AA^T = \begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 18 \end{pmatrix}
\]

We diagonalize \( AA^T \):

\[
det(AA^T - λI_2) = det \begin{pmatrix} 32 - λ & 0 \\ 0 & 18 - λ \end{pmatrix} = (32 - λ)(18 - λ) = 0 \rightarrow λ = 32, 18
\]

We can see that both \( A^T A \) and \( AA^T \) have the same eigenvalues. We calculate the eigenspaces.

\[
E(32) = Ker(AA^T - 32I_n) = Ker \begin{pmatrix} 0 & 0 \\ 0 & -14 \end{pmatrix} = \left\{ \begin{pmatrix} a \\ 0 \end{pmatrix}, a ∈ \mathbb{R} \right\}
\]

\[
E(18) = Ker(AA^T - 18I_n) = Ker \begin{pmatrix} 14 & 0 \\ 0 & 0 \end{pmatrix} = \left\{ \begin{pmatrix} 0 \\ a \end{pmatrix}, a ∈ \mathbb{R} \right\}
\]

An orthonomal matrix built from these eigenvectors is of the form:

\[
U = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \text{ with } a^2 = 1, b^2 = 1
\]

In this case, we can choose several orthonormal vectors. Eigenvectors from different eigenvalues are orthogonal to each other, but there is more than one possibility to define the unit vectors:

\[
\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \text{ all have unit length}
\]

To determine the sign of \( a \) and \( b \), recall that if \( v \) is an eigenvector of \( A^T A \) with eigenvalue \( λ \), then \( Av \) is an eigenvector of \( AA^T \); and to have the same eigenvalue we must define this eigenvector as:

\[
u = \frac{1}{\sqrt{λ}} Av
\]
Since the eigenvalues of $A^T A$ are $\lambda = 32, 18$, we then choose these two vectors $u$ as:

\[
v_1 = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \rightarrow A \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 4/\sqrt{2} \\ 0 \end{pmatrix} \rightarrow \\
\rightarrow u_1 = \frac{1}{4/\sqrt{2}} \begin{pmatrix} 4/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[
v_2 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \rightarrow A \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 3/\sqrt{2} \end{pmatrix} \rightarrow \\
\rightarrow u_2 = \frac{1}{3/\sqrt{2}} \begin{pmatrix} 0 \\ -3/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}
\]

Thus, we choose $U = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. As $AA^T$ is already a diagonal matrix, its diagonalization is trivial:

\[
AA^T = U D U^T \rightarrow \begin{pmatrix} 32 & 0 \\ 0 & 18 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 32 & 0 \\ 0 & 18 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

We finally have:

\[
A = U \Sigma V^T \rightarrow \begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4/\sqrt{2} & 0 \\ 0 & 3/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}
\]
3 Exercises

Ex. 1 — Find a singular value decomposition (SVD) for the matrix

\[ A = \begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix} \]

Ex. 2 — Find a singular value decomposition (SVD) for the matrix

\[ A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix} \]

Ex. 3 — Find a singular value decomposition (SVD) for the matrix

\[ A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \]

Ex. 4 — The Fibonacci sequence is an infinite sequence of integer numbers of the form:

\[ F = \{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots \} \]

Any element of the sequence \( F_n \) can be defined by the recurrence: \( F_n = F_{n-1} + F_{n-2} \), with initialization \( F_1 = 1, F_0 = 0 \). This recurrence can be expressed in a matrix form as follows:

\[ \begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} \]

So the matrix \( A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \) is usually called the Fibonacci matrix. Show that in the singular value decomposition (SVD) for \( A, A = UDV^T \), the matrix of singular values is

\[ S = \begin{pmatrix} \varphi & 0 \\ 0 & \varphi^{-1} \end{pmatrix} \]

where \( \varphi = \frac{1 + \sqrt{5}}{2} \) is the golden ratio.
**Ex. 5** — Consider the Fibonacci sequence in matrix form described above. The general form for any term in the series can be written as:

\[
\begin{pmatrix}
F_{n+1} \\
F_n
\end{pmatrix} = A^n \begin{pmatrix}
F_1 \\
F_0
\end{pmatrix}
\]

where \( A \) is the Fibonacci matrix described above. Use the singular value decomposition (SVD), \( A = UDV^T \), derived before to calculate \( A^n \). Use this decomposition to obtain the general form of an element of the sequence in terms of golden-ratio (\( \varphi \)):

\[
F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}
\]

where \( \varphi = \frac{1 + \sqrt{5}}{2} \)

Note: a matrix raised to an integer power \( n \) is the product of \( n \) matrices, e.g. \( A^3 = A \cdot A \cdot A \)
4 R practical

4.1 SVD

Singular value decomposition can be automatically done in R with the function `svd()`.

```r
# Define the matrix
> m <- matrix(c(4, -3, 4, 3), 2,2)

# Compute the SVD
> svd <- svd(m)

# svd() returns three objects:
# svd$d returns the list of the singular values:

> svd$d
[1] 5.656854 4.242641

# svd$u returns the left singular vectors matrix:

> svd$u
 [,1] [,2]
[1,] -1 0
[2,] 0 1

# svd$v returns the right singular vectors matrix

> svd$v
 [,1] [,2]
[1,] -0.7071068 -0.7071068
[2,] -0.7071068 0.7071068

# To have the right singular vector matrix transposed
# we used the command `La.svd()`

> svdt <- La.svd(m)
> svdt$vt
 [,1] [,2]
[1,] -0.7071068 -0.7071068
[2,] -0.7071068 0.7071068
```
In this particular case is the same as before
References
