Exercises – Vector spaces

1. Show that two vectors u, v are linear dependent if and only if there are two numbers $a, b \in \mathbb{R}$, such that $au+bv = 0$

2. Show that the set of vectors in the hyperplane $\mathbb{R}^n$ represented as $u=(u_1,\ldots,u_n)$ form a vector space.

3. Given the vectors $u=(0,0,-8,1)$, $v=(1,-8,0,7)$ in $\mathbb{R}^4$, find w such that $2u+v-3w=0$.

4. Given the vectors $u_1=(1,3,2,1)$, $u_2=(2,-2,-5,4)$, $u_3=(2,-1,3,6)$ and $v=(2,5,-4,0)$ $\in \mathbb{R}^4$. Write v as a linear combination of u_1, u_2, u_3. If it is not possible, say so.

5. Show that the set of all ordered pairs of real numbers is a vector space over $\mathbb{R}$.

6. Consider the set V of all polynomials of degree 2 (fixed degree). Is V a vector space? Justify the answer.

7. Consider the set V' of all polynomials of degree ≤ 2. Is V' a vector space? (compare with the previous exercise)

8. Given $S=\{ (1,-1), (2,1)\}$ a subset of $\mathbb{R}^2$, i.e. $S \subseteq \mathbb{R}^2$. Show that S is a set of generators of the vector space $\mathbb{R}^2$

9. Consider $S=\{ (1,1)\}$. Is S a generator of the vector space $\mathbb{R}^2$?

10. Show that $S=\{ (1,0,0), (0,1,0), (0,0,1)\}$ is a basis of the vector space $\mathbb{R}^3$

11. Given $v=(2,3,1) \in \mathbb{R}^3$, and $u_1=(2,0,1)$, $u_2=(0,1,1)$, $u_3=(0,0,2) \in \mathbb{R}^3$.
    a. Show that $S=\{u_1,u_2,u_3\}$ is a basis of $\mathbb{R}^3$,
    b. Find a representation of v in S
    c. Find a representation of the canonical basis $e_1=(1,0,0)$, $e_2=(0,1,0)$, $e_3=(0,0,1)$ in S.