Exercises - Linear maps

1. Given a vector space V, show that the Image of a linear map *f* over V, f(V) is also a vector subspace.

2. Given *f*:V→W a linear map, show that the Kernel of *f* = {set of all elements v∈V that map to the neutral element} is also a subspace.

3. Consider L the set of all points on a line through the origin of $\mathbb{R}^2$. Show that L is a subspace of $\mathbb{R}^2$. Recall that L can be represented as L={(x,y) ∈ $\mathbb{R}^2$ / ax+by=0, $\forall\, a, b \in \mathbb{R}$}

4. Consider the set of all points on a line that does not pass through the origin. Explain why this is not a subspace of $\mathbb{R}^2$.

5. Show f:$\mathbb{R} \rightarrow \mathbb{R}^2$, f(x) = (x,0) is a linear map.

6. Show that the constant map f:E → $\{k\}$, f(v) = k, $k \in \mathbb{R}$ is not a linear map.

7. Consider the set of polynomials of degree ≤ 2, $P_2 = \{v = ax^2 + bx + c; a, b, c \in \mathbb{R}\}$. Consider the following map of P_2 to $\mathbb{R}^3$:

   f: $P_2 \rightarrow \mathbb{R}^3$

   1 -> (1,0,0)

   x -> (0,1,0)

   x^2 -> (0,0,1)

   Show that *f* is an isomorphism.

8. Consider the vector space of functions of α : $G = \{a \cos\alpha + b \sin \alpha;\ a, b \in \mathbb{R}\}$. Show that G is isomorphic to $\mathbb{R}^2$ under the map: f: $G \rightarrow \mathbb{R}^2$, f($a \cos \alpha + b \sin \alpha$) = ($a$,$b$).

9. Consider the dilation map: f: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$, $f(x, y) = (sx, sy), s \in \mathbb{R}, s > 0$. Show that this is an automorphism of $\mathbb{R}^2$.

10. Consider the reflection map: f: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$, $f(x, y) = (x, -y)$. This map represents a reflection over the x axis. Show that this is an automorphism of $\mathbb{R}^2$.