Exercises –Matrix diagonalization

1. Show that det(AB) = det(A)det(B) for the two matrices:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},\ B = \begin{pmatrix} \alpha & b \\ \gamma & \delta \end{pmatrix}$$

2. Given the matrices $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$, calculate their determinant. What is their rank? What are the implications of these results?

3. For this problem assume that we know the following: If X is an m × m matrix, and Y is an m × n matrix and if 0 and I are zero and identity matrices of appropriate sizes, then $\det\begin{pmatrix} X & Y \\ 0 & I \end{pmatrix} = \det X$.

Given A an m×n matrix and B be an n×m matrix. Prove that $\det\begin{pmatrix} 0 & A \\ -B & I \end{pmatrix} = \det AB$. Hint: consider the product $\begin{pmatrix} 0 & A \\ -B & I \end{pmatrix}\begin{pmatrix} I & 0 \\ B & I \end{pmatrix}$

4. Consider the matrix $A = \begin{pmatrix} 1 & 5 & 2 \\ -1 & 1 & 4 \\ 0 & 2 & -1 \end{pmatrix}$

a) Calculate the cofactor matrix C

b) Calculate the adjoint matrix $Adj(A) = C^T$

c) Calculate the determinant for A

d) Calculate the inverse A^{-1}, using the previous results.

5. A matrix is called orthogonal if it is built by column vectors that are mutually perpendicular. A matrix is called orthonormal if it is orthogonal and all its column vectors have unit length. Show that the inverse of an $n \times n$ orthonormal matrix is its transpose. Hint: show that for $A^{T}A = I$ to be true, A has to be orthonormal.

6. Consider the matrix $A = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$:

a) Calculate its eigenvalues and eigenvectors.

b) Calculate P such that $P^{-1}AP$ is diagonal

7. Consider the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ -1 & 2 & 3 \end{pmatrix}$:

a) Calculate its eigenvalues and eigenvectors.

b) Calculate P such that $P^{-1}AP$ is diagonal

8. Consider the following linear map between polynomials of degree ≤ 1:

$f: P1 \to P1$. Where f: a+bx →(a+b) + (a+b)x. Calculate the eigenvalues and eigenvectors associated to this linear map.

9. Consider the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$:

Show that A is not diagonalizable. Hint: you can use the theorem that says that a square matrix of size *n* is diagonalizable if and only if it has *n* linearly independent eigenvectors. To do this, you can calculate the dimensions of the vector space spanned by each eigenvector.