

## Exercise

Problem: Does a patient have the disease or not

A patient takes a lab test and the result comes back positive.

The test returns a correct positive result in only 98% of the cases in which the disease is actually present and a correct negative result in only 97% of the cases in which the disease is not present.

Furthermore, 0.008 of the entire population have this disease.

$$P(c)$$

$$P(\bar{c})$$

$$P(+ | c)$$

$$P(+ | \bar{c})$$

$$P(- | c)$$

$$P(- | \bar{c})$$

$$P(c | +)?$$

# Bayes' Theorem

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$$P(c) = 0.008$$

$$P(\bar{c}) = 0.992$$

$$P(+ | c) = 0.98$$

$$P(+ | \bar{c}) = 0.03$$

$$P(- | c) = 0.02$$

$$P(- | \bar{c}) = 0.97$$

$$\text{Since } P(+ | c) + P(- | c) = \frac{P(+, c)}{P(c)} + \frac{P(-, c)}{P(c)} = \frac{1}{P(c)} P(c) = 1$$

And similarly for  $\bar{c}$

## Bayes' Theorem

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$$\begin{aligned} P(c|+) &= \frac{P(+|c)P(c)}{P(+)} = \frac{P(+|c)P(c)}{P(+|c)P(c) + P(+|\bar{c})P(\bar{c})} \\ &= \frac{0.980 \cdot 0.008}{0.980 \cdot 0.008 + 0.03 \cdot 0.992} = \frac{0.00784}{0.00784 + 0.02976} \end{aligned}$$

$$= 0.208$$

$$P(\bar{c}|+) = 0.792$$

Although the test is positive it is still not very likely that the patient has the disease

Homework: what is the probability to miss a case of disease?

## Exam question from 2016

The doctor selects you at random to have a blood test for swine flu, which is currently suspected to affect 1 in 10,000 people in Europe. The test is 99% accurate, in the sense that the probability of a false positive is 1%; and the probability of a false negative is zero. You test positive. What is the probability that you have swine flu?

However, you suddenly remembered that you recently went to a friend's wedding in Mexico, and it is known that 1 in 200 people who visited Mexico recently comes back with swine flu. Given the result of the test, what should your revised estimate be for the probability that you have the disease?